On the equations governing the second-order correlation functions for the velocity and the magnetic field of isotropic hydromagnetic turbulence in an incompressible fluid

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gravitational model proposed by the author (Stephenson 1969). Of perhaps greater significance is the fact that the dynamical nature of this gravitational model indicates a gravitational red-shift explanation of the varying red shifts that are observed to exist between individual members of related galaxies. Arp (1970) has shown that the different red shifts of the members of these galactic groups cannot arise solely from Doppler velocities and yet existing gravitational models do not predict sufficiently large gravitational red shifts to account for these observations.

An extremely simple way of testing this proposal is to check whether the observed red shifts of the individual members of a related galactic group are inverseley proportional to the squares of the radii of the individual galaxies; such a variation would be expected from a first-order approximation of the suggested gravitational model, if constant angular momentum is assumed for each galaxy.

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On the equations governing the second-order correlation functions for the velocity and the magnetic field of isotropic hydromagnetic turbulence in an incompressible fluid

Abstract. By applying the Smirnov method one derives the equations describing the correlation functions of the velocity and the magnetic field for an isotropic non-homogeneous hydromagnetic turbulence in an incompressible conducting fluid.

The statistical treatment of the theory of hydromagnetic turbulence involves an incomplete set of equations, whose number is less than the number of unknown functions (i.e. of all sorts of correlations). Supplementary arguments are to be imposed in order to obtain a complete set of equations.

Chandrasekhar (1955) formulated a deductive theory of isotropic homogeneous stationary hydromagnetic turbulence and obtained the equations for the second-order correlation functions of the velocity and the magnetic field respectively.

Lee (1965) developed a new formulation of the theory of stationary hydromagnetic turbulence as a generalization of Wyld (1961) theory of ordinary turbulence. This formulation involves terms describing the external force.

Recently Smirnov (1970) obtained the equation for the second-order correlation functions of the velocity in ordinary isotropic non-stationary turbulence. Using a hydromagnetic generalization of the Smirnov method we deduced a pair of equations for the second-order correlation functions of the velocity and the magnetic field in a non-stationary isotropic hydromagnetic turbulence.

The equations of motion for an incompressible conducting fluid in a magnetic field are:

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = \nu \Delta u_i - \frac{\partial \omega}{\partial x_i} + f_i \tag{1}$$

and

$$\frac{\partial h_i}{\partial t} + \frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \lambda \Delta h_i \tag{2}$$

where u_i ($i = 1, 2, 3$) denote the components of the velocity, h_i the components of the magnetic field divided by $(4\pi\rho/\mu)^{1/2}$, f_i the components of the external force per unit mass, ρ the density, p the pressure and μ, ν, σ are the coefficients of magnetic permeability, kinematic viscosity and electrical conductivity respectively and

$$\lambda = \frac{1}{4\pi\mu\sigma}, \qquad \omega = \frac{p}{\rho + \frac{1}{2}|\boldsymbol{h}|^2}.$$

We consider the following correlation tensors:

$$\begin{aligned}
&Q_{ij} = \langle u_i(\boldsymbol{r}', t')u_j(\boldsymbol{r}'', t'')\rangle = \langle u_i' u_j''\rangle, \quad H_{ij} = \langle h_i' h_j''\rangle, \quad G_{ij} = \langle f_i' u_j''\rangle, \\
&P_{ij} = \langle u_i' u_j' \omega''\rangle, \quad \Pi_{ij} = \langle h_i' h_j' \omega''\rangle, \quad T_{ij,k} = \langle u_i' u_j' u_k''\rangle, \\
&S_{ij,k} = \langle h_i' h_j' u_k''\rangle, \quad \acute{I}_{ij,k} = \langle u_i' u_j' f_k''\rangle, \quad F_{ij,k} = \langle (h_i' u_j' - u_i' h_j') h_k''\rangle, \\
&Q_{ij,kl} = \langle u_i' u_j' u_k'' u_l''\rangle, \quad H_{ij,kl} = \langle h_i' h_j' h_k'' h_l''\rangle, \\
&R_{ij,kl} = \langle (h_i' u_j' - u_i' h_j')(h_k'' u_l'' - u_k'' h_l'')\rangle
\end{aligned} \tag{3}$$

where $\boldsymbol{r}'$ and $\boldsymbol{r}''$ are the radius vectors of two neighbouring points; t', t'' are two instants of time and the angular brackets denote ensemble averages.

By applying Smirnov's method to equations (1) and (2) we obtain the following equations:

$$\begin{aligned}
\frac{\partial}{\partial r}\left(\frac{\partial}{\partial t''} - \nu D_5\right)\left(\frac{\partial}{\partial t'} - \nu D_5\right)Q = 2Q\frac{\partial}{\partial r}D_5 Q + 2H\frac{\partial}{\partial r}D_5 H + \frac{\partial}{\partial r}\left(\frac{\partial}{\partial t''} - \nu D_5\right)G \\
+ \frac{\partial}{\partial r}\left(5 + r\frac{\partial}{\partial r}\right)\acute{I}
\end{aligned} \tag{4}$$

$$\left(\frac{\partial}{\partial t''} - \lambda D_5\right)\left(\frac{\partial}{\partial t'} - \lambda D_5\right)H = 2QD_5 H + 2HD_5 Q + 2\frac{\partial Q}{\partial r}\frac{\partial H}{\partial r} \tag{5}$$

where $Q(r, t', t'')H(r, t', t'')$, $G(r, t', t'')$, $\acute{I}(r, t', t'')$ are the scalars defined by the correlation tensors Q_{ij}, H_{ij}, etc., $r = |\boldsymbol{r}'' - \boldsymbol{r}'|$ and $D_5 = \partial^2/\partial r^2 + (4/r)\partial/\partial r$ is the five-dimensional Laplacian operator.

If the hydromagnetic turbulence is stationary, homogeneous and isotropic, the scalars Q, H, etc. depend on r and $\tau = t'' - t'$. By splitting the correlation tensors in even and odd parts with respect to the time τ, we find:

$$-\frac{\partial}{\partial r}\left(\frac{\partial^2}{\partial \tau^2}-\nu^2 D_5{}^2\right)Q = 2Q\frac{\partial}{\partial r}D_5 Q+2H\frac{\partial}{\partial r}D_5 H+\frac{\partial}{\partial r}\frac{\partial G_1}{\partial \tau}-\nu\frac{\partial}{\partial r}D_5 G_2+\frac{\partial}{\partial r}\times\left(5+r\frac{\partial}{\partial r}\right)I_2' \tag{6}$$

$$-\left(\frac{\partial^2}{\partial \tau^2}-\lambda^2 D_5{}^2\right)H = 2QD_5 H+2HD_5 Q+2\frac{\partial Q}{\partial r}\frac{\partial H}{\partial r} \tag{7}$$

where $G_1+G_2 = G$; G_1, G_2 are the odd and even parts respectively, and I_2' is the even part of I'.

One immediately remarks that if we leave out the external force terms, the equations (6), (7) reduce to Chandrasekhar's equations. We conclude this letter by pointing out that the equations deduced by us generalize the well-known equations for the second-order correlation functions for the velocity and the magnetic field.

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The intensity fluctuation distribution of laser light

Abstract. An experimental investigation of laser noise by measurement of photon-counting distributions over a range of sample times and pumping levels is reported. These results are compared with theoretical predictions and show good agreement. In addition a simple approximate expression for the second moment of the intensity fluctuations is given.

In a recent paper Lax and Zwanziger (1970) computed the intensity fluctuation distribution of integrated laser light near threshold. They remarked that detailed measurements of $p(m, T)$, the photon counting distribution for arbitrary sample time T, had not yet been reported though several workers have made measurements with short sample times (e.g. Freed and Haus 1966, Armstrong and Smith 1965, Arecchi *et al.* 1967, Chang *et al.* 1967, Pike 1969). In this letter we present a set of such detailed noise measurements for a Spectra Physics model 119 single-mode gas laser. Contact is made with the above computations through the factorial moments of $p(m, T)$, which are the actual moments of the intensity fluctuation distribution. A simple approximate analytic expression for the second factorial moment is given which gives results indistinguishable from the full theory. We compare the experimental values for these moments with detailed results of Lax and Zwanziger (1970).